Culminating Task: Are You Positive?

MCC9-12.S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. *

MCC9-12.S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. *

MCC9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. *

MCC9-12.S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *

MCC9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. *

We know that modern medicine is rich with biology, chemistry and countless other branches of science. While mathematics does not make a lot of headlines for changing the way we think about our health and well-being, there are many ways in which mathematics is informing and improving the way scientists and doctors approach the world of medicine.

Consider the following data for a group of 1000 women. Of these women, 8 are known to have breast cancer. All 1000 undergo the same test to determine whether or not they have breast cancer, and 77 test positive. 7 of the 8 that have breast cancer test positive and 70 of those that do not have breast cancer test positive.

1. Based on the data presented above, do you think the test for breast cancer is effective at identifying women who have breast cancer? Why or why not?
2. Organize the data using the table below.

<table>
<thead>
<tr>
<th>Women with Breast Cancer</th>
<th>Test Positive</th>
<th>Test Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Women without Breast Cancer</td>
<td>Test Positive</td>
<td>Test Negative</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Find the following probabilities:

   a. The probability that a woman tests positive given that she has breast cancer.

   b. The probability that a woman tests positive given that she does not have breast cancer.

4. Determine whether or not the events of having breast cancer and testing positive for breast cancer are independent. Show all relevant calculations.

The analysis you have done so far should seem straightforward. Testing for breast cancer is successful at identifying those women who have it. It is reassuring to know that testing positive for breast cancer is not independent of having breast cancer, as this seems to indicate that screening for breast cancer is an effective way to identify when a woman actually has breast cancer.

There is one aspect of this analysis that needs further inspection. The probabilities that you have found have the condition that a woman does or does not have breast cancer. In reality, a woman knowing this before getting tested is highly unlikely. The point of getting tested is to find out!
While the results you have found seem sound, it will be good to find the probabilities from a more realistic standpoint.

5. Now you will look through the lens of a woman who tests positive for breast cancer.
   a. Find the probability that a woman has breast cancer given that her test result is positive.

   b. What seems strange about this result to you?

   c. Compare this probability to what you calculated in question 3a. What is causing these probabilities to be so different?

6. Let’s also look through the lens of a woman who tests negative.
   a. Find the probability that a woman does not have breast cancer given that her test result is negative.

   b. What does this result indicate?

7. This task focused specifically on medical tests for breast cancer. It is not a stretch to say that the efficacy of most medical tests is similar to that of what you have investigated here. Write a paragraph that discusses the use and effectiveness of medical tests in regards to the probability theory that underlies them.